

Quasiperiodic time dependent current in driven superlattices: distorted Poincaré maps and strange attractors

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Abstract

Intriguing routes to chaos have been experimentally observed in semiconductor superlattices driven by an ac field. In this work, a theoretical model of time dependent transport in ac driven superlattices is numerically solved. In agreement with experiments, distorted Poincaré maps in the quasiperiodic regime are found. They indicate the appearance of very complex attractors and routes to chaos as the amplitude of the AC signal increases. Distorted maps are caused by the discrete well-to-well jump motion of a domain wall during spiky high-frequency self-sustained oscillations of the current.

Nonlinear dynamics of weakly coupled semiconductor superlattices (SLs) driven by dc and ac biases has been the research topic of many experimental and theoretical works [1–7]. The nonlinearity manifests itself in many physical situations as for instance, in their transport properties under finite DC or AC bias. Under appropriate DC voltage bias, the current through the SL displays natural oscillations caused by creation, motion and annihilation of domain walls (DWs) in the SL [4,6]. The observed oscillation frequencies range from kHz to GHz. Superimposed on a smooth current oscillation, there appear faster current spikes whose frequencies are typically in the GHz regime [6]. Each spike reflects the well-to-well motion of DWs causing the self-oscillation, and therefore their frequency is related to the characteristic tunneling time. Theory and numerical simulations show that DW may be charge monopoles or dipoles [7,8], although experimental evidence shows that DW are charge monopoles for self-oscillations observed in the available samples [6]. AC driven SLs display a rich dynamical behavior including quasiperiodic and chaotic oscillations with nontrivial spatial structure [9–13].

Many studies fix the frequency of the AC drive as the golden mean number $(1 + \sqrt{5})/2 \approx 1.618$ times the frequency of the natural oscillations (i.e., the frequency ratio is an irrational number hard to approximate by rational numbers), which is convenient to obtain complex dynamical behavior [14]. In this case, the system presents a rich power spectrum, a complex bifurcation diagram and different routes to chaos including quasiperiodicity, frequency locking, etc. [9–13]. First return or Poincaré maps are used to analyze unambiguously the underlying attractors [13,14]. In the quasiperiodic case, Poincaré maps usually consist of smooth loops, whereas they are a set of discrete points in the case of frequency locking. More exotic Poincaré maps resembling distorted double loops in the quasiperiodic case have been experimentally observed in middle of the second plateau of the I–V characteristic of a SL [12,13]. At the onset of this plateau, Poincaré maps are smooth and not distorted. The origin of distorted maps was not understood at the time of their observation, although disorder and sample imperfections were invoked [12]. Luo et al [13] showed that a combination of signals with different frequency was needed in order to reproduce experimentally observed

distorted double layer Poincaré maps. The origin of this combination was not ascertained in that work.

The aim of the present work is to establish theoretically that high-frequency current spikes of the self-oscillations give rise to these exotic Poincaré maps. In turn, current spikes are due to the well-to-well motion of DW during each period of the self-oscillations. Thus distorted Poincaré maps reflect DW motion in AC driven SLs.

Theoretical studies of nonlinear effects in weakly coupled SL are based in microscopic modelization of the sequential tunneling current between adjacent wells [5,7]. Early models were discrete drift models with a phenomenological fitting of tunneling current and boundary conditions [4]. More sophisticated models have included the electrostatics at the contacts in a self-consistent framework [5,8]. Depending on SL configuration and contact doping, we know that undriven self-oscillations are due to recycling of either monopole DWs or dipole waves [8]. In the latter case, a dipole wave consists of two DWs, one corresponding to a charge accumulation layer and another one corresponding to a depletion layer. Between these DWs there is a high-field region encompassing several wells, and the whole dipole wave travels through the SL. Recycling of such dipole waves gives rise to current self-oscillations similar to the well-known Gunn effect in bulk semiconductors [15].

In this paper we have analyzed the tunneling current through a DC+AC biased SL by means of the microscopic model described in [5,8]. We have considered a 50-period SL consisting of 13.3 nm GaAs wells and 2.7 nm AlAs barriers, as described in [6]. Doping in the wells and in the contacts are $N_w = 2 \times 10^{10} \text{ cm}^{-2}$ and $N_c = 2 \times 10^{18} \text{ cm}^{-3}$ respectively. With these doping values, self-oscillations are due to recycling of monopole DWs. We choose not to analyze the original 9/4 sample where the experiments were performed [12]. In this sample, the second plateau of the I-V characteristics ends at a resonant peak due to $\Gamma - X$ tunneling. In the simpler SL configuration chosen here, the X point does not contribute to the peaks at the first and second plateaus. We analyze the sequential tunneling current [8] with an applied voltage, $V(t) = V_{AC}(t) + V_{DC}$, where $V_{AC}(t) = V_{AC} \sin(2\pi f_{AC}t)$. The applied AC frequency f_{AC} has been set to the golden mean times the natural frequency and

is very small compared with typical energy scales of the system. Thus the AC potential modifies adiabatically the potential profile of the SL and our sequential tunneling model holds [5,8].

Fig. 1 shows the evolution of the current through the SL, its Fourier spectrum and its Poincaré map for $V_{DC} = 4.2$ V and $V_{AC} = 19$ mV. These values correspond to the onset of the second plateau of the I–V characteristic curve. The current trace of Fig. 1(a) is quasiperiodic and does not present observable superimposed high-frequency oscillations (spikes). The natural oscillation near the onset of the plateau is caused by monopole recycling very close to the collector contact. Thus the DW does not move over many wells and the current trace does not present an appreciable number of spikes. In the power spectrum of Fig. 1(b) there are contributions coming from the fundamental frequency $f_0=39$ MHz, the frequency of the applied AC field f_{AC} , the combination of both and their higher harmonics. The Poincaré map depicted in Fig. 1(c) is a smooth loop with a nontrivial double layer structure indicating quasiperiodic oscillations. If we increase the DC bias up to the middle of the second plateau, $V_{DC} = 5.1$ V, the undriven oscillation is due to recycling of monopole DWs which propagate periodically through part (approximately 40%) of the structure and disappear at the collector. A similar motion is observed in the AC driven case. The corresponding current trace may show high frequency spikes depending on the chosen initial field profile. A flat initial field profile gives a spiky current trace until the latter settles to the stable oscillation (over which there are no appreciable spikes); see the earlier part of Fig. 2(a). The power spectrum and Poincaré maps corresponding to this case and to the previous one (Figs. 1(b) and (c)) are markedly different. Fig. 2(b) shows that the power spectrum of the case with spikes has a greater harmonic content than that of the case without spikes (Fig. 1), presenting many peaks corresponding to higher harmonics of the fundamental frequencies (i.e., the low fundamental frequency oscillations, $f_0 = 33$ MHz, and the spikes, 500 MHz), the applied frequency and combinations thereof. The corresponding Poincaré map consists of a distorted loop with a double layer structure which shows a strong similarity with previous experiments [12,13]; see Fig. 2(c). From these numerical observations, we conclude

that distorted Poincaré maps are linked to spiky current traces, even if such traces change to smooth ones after a transient.

The previous conclusion may be reinforced if we change the emitter doping so that undriven self-oscillations are caused by dipole waves, for which the current traces are more spiky [8]. Thus we carried out numerical simulations with a smaller contact doping, $N_c = 2 \times 10^{16} \text{ cm}^{-3}$. The frequency of the natural oscillation is now reduced to 4 MHz and the AC intensity is 2 mV. The current at the middle of the first plateau ($V_{DC}=1.5 \text{ V}$) is shown in Fig. 3(a). At finite time, $I(t)$ presents dipole-like oscillations and superimposed finite amplitude spikes. The Poincaré map, Fig. 3(c), is much more complicated than in the previous case, showing three well defined distorted loops. Since loops in the Poincaré map are due to combination of strong enough signals of different frequencies [13], the greater strength of the high-frequency spikes gives rise to the additional loop structure and higher harmonic content (Fig. 3(b)).

As we mentioned above, the double layer structure of Poincaré maps indicates nontrivial attractors for the quasiperiodic case. We have calculated the multifractal dimensions of the attractors, D_q , for the three cases discussed above; see Fig. 4. In all cases, they correspond to strange attractors [10,16,17] whose D_q presents the knee-like structure typical of chaotic attractors with multifractal dimensions [10]. A detailed discussion of the attractor dimensions will be presented elsewhere.

In summary, we have analyzed theoretically the time dependent current through a AC driven SL. We have characterized intriguing Poincaré maps of the quasiperiodic oscillations. The strange attractors which define these Poincaré maps have their physical origin in the complex dynamics of the domain wall. We have shown that distorted loops in the Poincaré maps are related to the presence of high frequency spikes in the current traces. Their frequencies combine with the AC frequency and the low natural frequency to yield a richer power spectrum. Since spikes are associated to extended motion of the DW, we can understand why distorted Poincaré maps are observed at the middle of a I-V plateau and not at its beginning, where the monopole moves over a too small part of the SL. The case of

natural self-oscillations due to dipole recycling is different. There the DW of the dipole are generated at the emitter contact and move over the whole SL. In such a case, spikes are more prominent than in the monopole case, and a higher harmonic content and distorted Poincaré maps appear. Our results may help explaining recent experimental evidence showing complex Poincaré maps and intriguing routes to chaos [13].

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REFERENCES

- [1] H.T. Grahn, R.J. Hang, W. Muller, and K. Ploog, Phys. Rev. Lett. **67**, 1618 (1991).
- [2] F. Prengel, A. Wacker, and E. Schöll, Phys. Rev. B **50**, 1705 (1994).
- [3] H.T. Grahn, J. Kastrup, K.H. Ploog, L.L. Bonilla, J. Galan, M. Kindelan, and M. Moscoso, Jpn. J. Appl. Phys., Part 1 **34**, 4526 (1995).
- [4] L. L. Bonilla, J. Galan, J.A. Cuesta, F.C. Martinez, and J.M. Molera, Phys. Rev. B **50**, 8644 (1994).
- [5] R. Aguado, G. Platero, M. Moscoso, and L.L. Bonilla, Phys. Rev. B **55**, R16053 (1997).
- [6] J.Kastrup, R. Hey, K.H. Ploog, L.L. Bonilla, J. Galan, M. Kindelan, and M. Moscoso, Phys. Rev. B **55**, 2476 (1997); J.W. Kantelhardt, H.T. Grahn, K.H. Ploog, M. Moscoso, A. Perales, and L.L. Bonilla, Physica Status Solidi B **204**, 500 (1997).
- [7] A. Wacker, in *Theory and transport properties of semiconductor nanostructures*, edited by E. Schöll. (Chapman and Hall, New York, 1998), chapter 10.
- [8] D. Sánchez, M. Moscoso, L.L. Bonilla, G. Platero, and R. Aguado, Phys Rev B, **60**, 4489 (1999).
- [9] O.M. Bulashenko, M.J. Garcia, and L.L. Bonilla, Phys. Rev. B **53**, 10 008 (1996).
- [10] O.M. Bulashenko, K.J. Luo, H.T. Grahn, K.H. Ploog, and L.L. Bonilla, Phys. Rev. B **60**, 5694 (1999).
- [11] Y. Zhang, J. Kastrup, R. Klann, K.H. Ploog, and H.T. Grahn, Phys. Rev. Lett. **77**, 3001 (1996).
- [12] K.J. Luo, H.T. Grahn, K.H. Ploog and L.L. Bonilla, Phys. Rev. Lett. **81**, 1290 (1998).
- [13] K. J. Luo, H.T. Grahn, S.W. Teitworth and K.H. Ploog, Phys. Rev. B. **58**, 12613 (1998).

- [14] E. Ott, *Chaos in dynamical systems* (Cambridge University Press, New York, 1993).
- [15] L.L. Bonilla, I.R. Cantalapiedra, G. Gomila, and J.M. Rubi, Phys. Rev. E **56**, 1500 (1997).
- [16] G. Grebogi, E. Ott, S. Pelikan, and J.A. Yorke, Physica D **13**, 261 (1984).
- [17] W.L. Ditto, M.L. Spano, H.T. Savage, S.N. Rauseo, J.F. Heagy and E. Ott, Phys. Rev. Lett. **65**, 533 (1990).

FIGURES

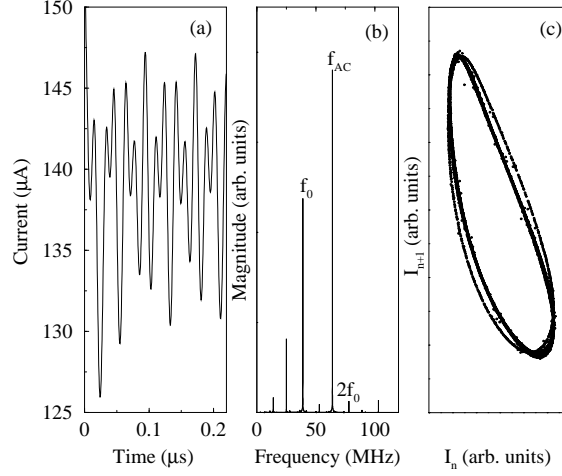


FIG. 1. (a) $I(t)$ for $V_{DC} = 4.2$ V, $f_0 = 39$ MHz, $V_{AC} = 19$ mV. Spikes are not resolved. (b) Power spectrum. Notice that higher harmonics of the fundamental frequency are barely formed. (c) Poincaré map, constructed by plotting the current at the $(n+1)$ st AC period versus the current in the preceding period.

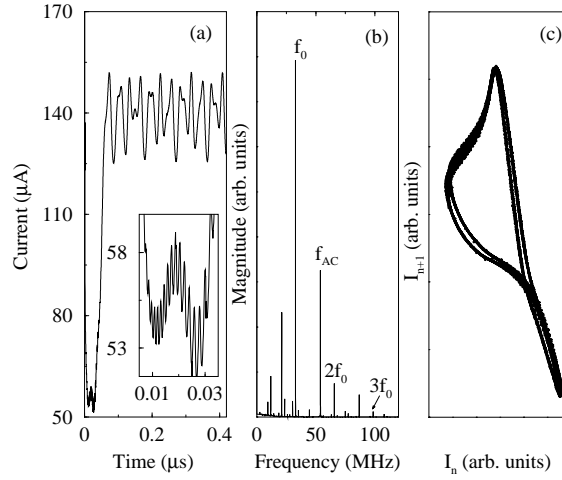


FIG. 2. (a) $I(t)$ for $V_{DC} = 5.1$ V, $f_0 = 33$ MHz. Spikes are present in the transient regime (see inset). (b) Power spectrum. Some higher harmonics of f_0 can be observed. (c) Poincaré map. Notice that the loop is somewhat distorted due to the presence of higher harmonics in (b).

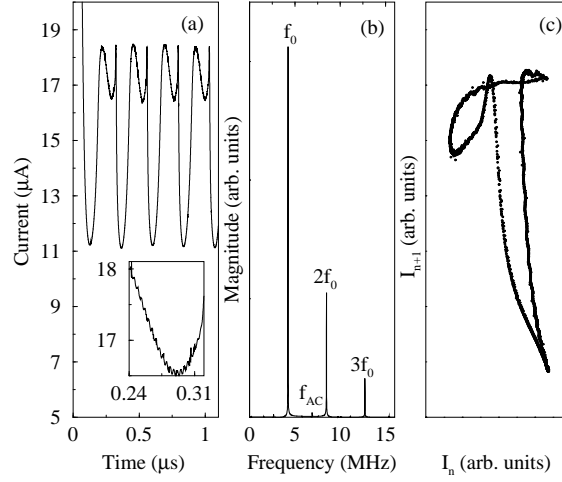


FIG. 3. (a) $I(t)$ for $V_{DC} = 1.5$ V, $f_0 = 4$ MHz, $V_{AC} = 2$ mV. Spikes are superimposed on the current throughout the signal (see inset). (b) Power spectrum. Higher harmonics of f_0 contribute with a finite amplitude to the power spectrum. (c) Poincaré map. The distortion is greater than in Fig. 2(c) (see text).

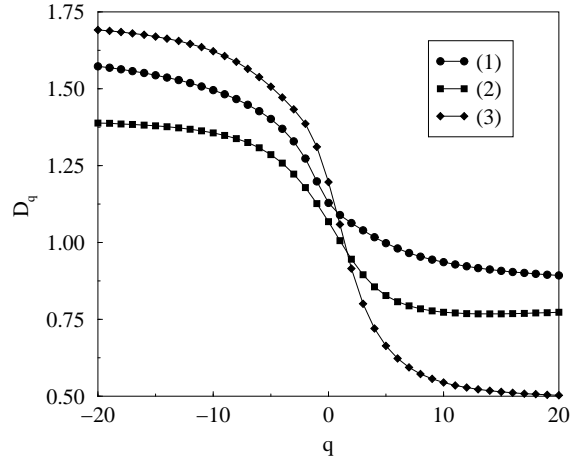


FIG. 4. Multifractal dimension. The labels (1), (2) and (3) correspond to the attractors in Figs. 1(c), 2(c) and 3(c), respectively.